FAR BEYOND

MAT122

Definite Integral

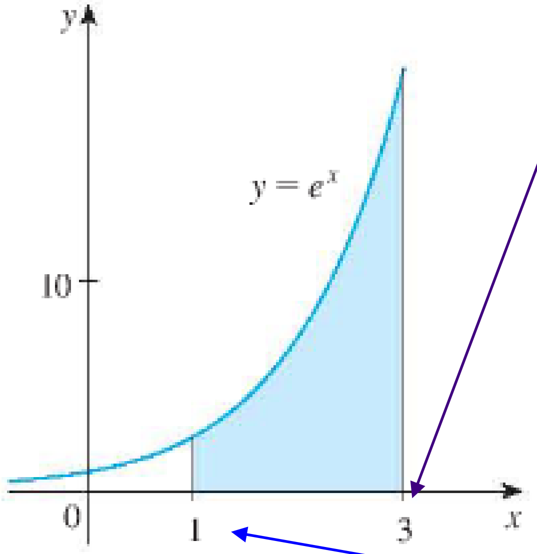
Stony Brook University

Definite Integration Notation

Definite Integral

$$\int_a^b f(x)\,dx$$

$x = a$ and $x = b$ are the bounds of the integration



$b = 3$

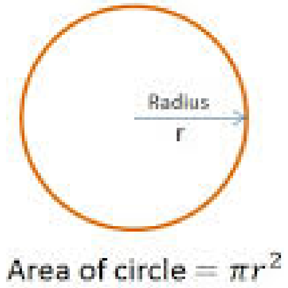
$$\int_1^3 e^x dx$$

$a = 1$

Evaluating Integrals using Area of Circle

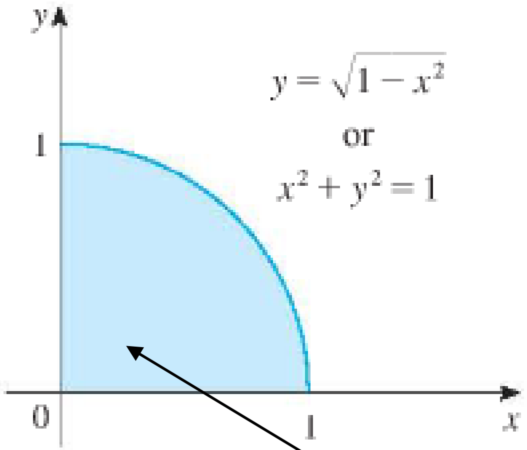
ex. Evaluate the following integral *by interpreting in terms of area.*

Area of a Circle:



$$\int_0^1 \sqrt{1-x^2}\,dx$$

$f(x)=\sqrt{1-x^2}$ is a quarter of a circle.



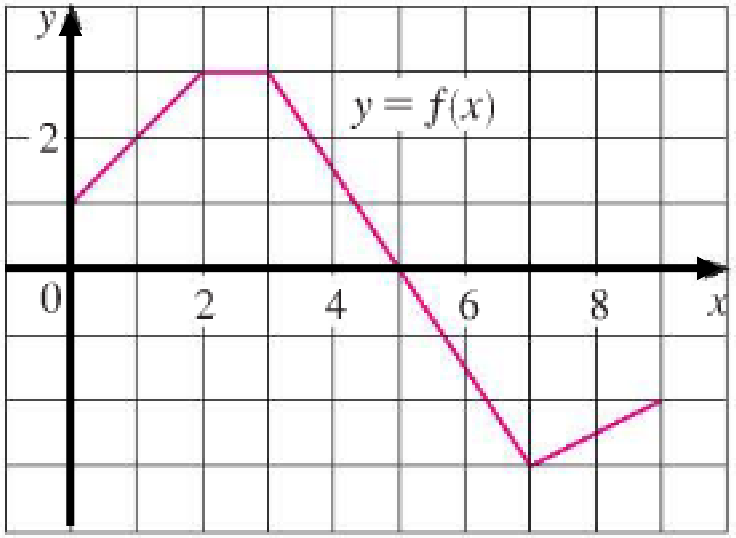
$r = 1$

$$A=\frac{1}{4}\pi r^2$$

$$=\frac{1}{4}\pi(1)^2 = \boxed{\frac{\pi}{4}}$$

Evaluating Integrals using Standard Shapes

ex. Evaluate the integrals of f using areas of standard shapes.



$\int_0^2 f(x)dx$

$= \boxed{4}$

$\int_2^3 f(x)dx \qquad = \boxed{3}$

$\int_3^5 f(x)dx \qquad = \boxed{3}$

$\int_5^7 f(x)dx \qquad = \boxed{-3}$

Do $\int_7^9 f(x)dx \qquad \boxed{}$

Then $\int_0^9 f(x)dx =$

$= \boxed{2}$

Indefinite vs. Definite Integral

Recall: $\boxed{\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C}$

The *indefinite* integral has no bounds.

There is a family of solutions.

ex. $\int 4x^3\,dx = \frac{\cancel{4}x^4}{\cancel{4}} + C \quad = x^4 + C$

The ***definite*** integral has bounds from $x = a$ to $x = b$. There is a single solution.

$$\boxed{\begin{array}{l}\text{If } f \text{ is continuous on } a \le x \le b \text{ and } F'(x) = f(x) \text{ then} \\ \qquad \int_a^b f(x)\,dx = F(b) - F(a).\end{array}}$$

ex. $\int_1^3 4x^3\,dx = \underbrace{x^4}_{F(x)}\Big|_{1\ a}^{3\ b} = F(\overset{b}{3}) - F(\overset{a}{1}) = 3^4 - 1^4$

$= 81 - 1$

$= \boxed{80}$

Definite Integral w/Polynomial

$$\boxed{\int_m^n ax^n\,dx = \left.\frac{ax^{n+1}}{n+1}\right|_m^n}$$

ex. Evaluate $\int_0^1 \left(1+\frac{1}{2}u^4+\frac{2}{5}u^9\right)du$

$$= \left.\left(u+\frac{1}{2}\cdot\frac{u^5}{5}+\frac{2}{5}\cdot\frac{u^{10}}{\cancelto{5}{10}}\right)\right|_0^1$$

$$= \left.\left(u+\frac{u^5}{10}+\frac{u^{10}}{25}\right)\right|_0^1 = F(b)-F(a)$$

$$= F(1)-F(0)$$

$$= \left(1+\frac{1^5}{10}+\frac{1^{10}}{25}\right)-\cancelto{0}{\left(0+\frac{0^5}{10}+\frac{0^{10}}{25}\right)}$$

$$= 1+\frac{1}{10}+\frac{1}{25}$$

get common denominator

$$= 1\cdot\frac{10\cdot 25}{10\cdot 25}+\frac{1}{10}\cdot\frac{25}{25}+\frac{1}{25}\cdot\frac{10}{10} = \frac{250+25+10}{250} = \frac{285}{250} = \frac{\cancel{5}\cdot 57}{\cancel{5}\cdot 50} = \boxed{\frac{57}{50}}$$

Definite Integral w/Radical

$$\int_m^n ax^n\,dx = \left.\frac{ax^{n+1}}{n+1}\right|_m^n$$

ex. Evaluate $\int_1^{18} \sqrt{\frac{3}{z}}\,dz$ put in ax^n format

$$= \int_1^{18} \frac{\sqrt{3}}{\sqrt{z}}\,dz$$

$$= \int_1^{18} \sqrt{3}z^{-1/2}\,dz$$ ready to integrate

$$= \sqrt{3}\cdot\left.\frac{z^{+1/2}}{\frac{1}{2}}\right|_1^{18}$$

$$= \sqrt{3}\cdot\left.\frac{2}{1}\cdot\sqrt{z}\right|_1^{18}$$

$$= 2\sqrt{3}\cdot\sqrt{z}\Big|_1^{18} = 2\sqrt{3}\left(\sqrt{18}-\sqrt{1}\right) = \boxed{2\sqrt{3}\left(3\sqrt{2}-1\right)}$$

$$\sqrt{18} = \sqrt{9\cdot 2}$$
$$= \sqrt{9}\sqrt{2}$$
$$= 3\sqrt{2}$$

Definite Integral with Exponential

ex. Evaluate $\int_0^1 e^x \, dx = e^x \Big|_0^1 = e^1 - e^0$

$= \boxed{e - 1}$

$$(e^x)' = e^x$$

$$\int_m^n e^x dx = e^x \Big|_m^n$$

ex. Find $\dfrac{d}{dx}\left(\dfrac{e^{3x}}{3}\right) = \dfrac{d}{dx}\left(\dfrac{1}{3}e^{\overset{u}{3x}}\right) = \dfrac{1}{3} e^u \cdot u'$

$= \dfrac{1}{\cancel{3}} e^{3x} \cdot \cancel{3}$

$= \boxed{e^{3x}}$

$u = 3x$
$u' = 3$

ex. Evaluate $\int e^{3x} \, dx = \dfrac{e^{3x}}{3} + C$

$$\int e^u dx = \frac{e^u}{u'} + C$$

$$\int_m^n e^u dx = \frac{e^u}{u'} \Bigg|_m^n$$